

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Pearson Edexcel		Centre Number	Candidate Number
Level 3 GCE			
Thursday 16 May 2019			
Afternoon		Paper Reference 8FM0-22	
Further Mathematics			
Advanced Subsidiary Further Mathematics options 22: Further Pure Mathematics 2 (Part of option A only)			
You must have: Mathematical Formulae and Statistical Tables (Green), calculator			Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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1. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 2 & 2 \end{pmatrix}$$

(a) find the characteristic equation for the matrix $\mathbf{A}$, simplifying your answer.

(2)

(b) Hence find an expression for the matrix $\mathbf{A}^{-1}$ in the form $\lambda\mathbf{A} + \mu\mathbf{I}$, where λ and μ are constants to be found.

(3)

$$(a) \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} 3-\lambda & 2 \\ 2 & 2-\lambda \end{vmatrix} \quad \lambda\mathbf{I} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$0 = (3-\lambda)(2-\lambda) - (2 \times 2) \quad (1)$$

$$0 = 6 - 5\lambda + \lambda^2 - 4$$

$$0 = \lambda^2 - 5\lambda + 2 \quad (1)$$

$$(b) \mathbf{A}^2 - 5\mathbf{A} + 2\mathbf{I} = \mathbf{0} \quad (1) \leftarrow \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \text{ which is the equivalent of } 1 \text{ in matrix calculations}$$

$$\mathbf{A} - 5\mathbf{I} + 2\mathbf{A}^{-1} = \mathbf{0} \quad (1)$$

$$2\mathbf{A}^{-1} = 5\mathbf{I} - \mathbf{A}$$

$$\mathbf{A}^{-1} = \frac{5}{2}\mathbf{I} - \frac{1}{2}\mathbf{A} \quad (1)$$

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Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 5 marks)



2. (i) Determine **all** the possible integers a , where $a > 3$, such that

$$15 \equiv 3 \pmod{a}$$

$$\hookrightarrow \text{congruence: } 15 \pmod{a} = 3 \pmod{a} \quad (2)$$

- (ii) Show that if p is prime, x is an integer and $x^2 \equiv 1 \pmod{p}$ then either

$$x \equiv 1 \pmod{p} \quad \text{or} \quad x \equiv -1 \pmod{p} \quad (3)$$

- (iii) A company has £13 940 220 to share between 11 charities.

Without performing any division and showing all your working, decide if it is possible to share this money equally between the 11 charities.

(2)

$$(i) \quad 4, 6, 12$$

$$3 \pmod{4} = 3 \quad 15 \pmod{4} = 3 \quad \checkmark$$

$$3 \pmod{5} = 3 \quad 15 \pmod{5} = 0 \quad \times$$

$$3 \pmod{6} = 3 \quad 15 \pmod{6} = 3 \quad \checkmark$$

$\vdots$

$$3 \pmod{12} = 3 \quad 15 \pmod{12} = 3$$

When $a > 15$, $3 \pmod{a}$ will always be 3 and $15 \pmod{a}$ will always be 15, so they are not equal.

$$(ii) \quad \text{Given } x^2 \equiv 1 \pmod{p}, \text{ then } x^2 - 1 \text{ must be divisible by } p \quad (1)$$

$$\therefore (x-1)(x+1) \text{ is divisible by } p.$$

$$\text{Since } p \text{ is prime, either } (x-1) \text{ OR } (x+1) \text{ is divisible by } p. \quad (1)$$

$$\therefore x \equiv 1 \pmod{p} \text{ or } x \equiv -1 \pmod{p} \quad (1)$$

$$(iii) \quad 1 - 3 + 9 - 4 + 0 - 2 + 2 - 0 = 3 \quad (2) \quad \text{OR} \quad \begin{array}{l} \text{Sum (odd)} = 12 \\ \text{Sum (even)} = 9 \\ 12 - 9 = 3 \end{array}$$

$\hookrightarrow$ select a divisibility test.

3 is not divisible by 11, therefore it is not possible to share the money equally between the 11 charities. (1)

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Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



3. A curve C in the complex plane is described by the equation

$$|z - 1 - 8i| = 3|z - 1|$$

(a) Show that C is a circle, and find its centre and radius.

(4)

(b) Using the answer to part (a), determine whether $z = 3 - 3i$ satisfies the inequality

$$|z - 1 - 8i| \geq 3|z - 1|$$

(2)

(c) Shade, on an Argand diagram, the set of points that satisfies both

$$|z - 1 - 8i| \geq 3|z - 1| \quad \text{and} \quad 0 \leq \arg(z + i) \leq \frac{\pi}{4}$$

(4)

(a) $z = x + yi$ and $|z - z_1| = r$ is a circle centre z_1 .

$$(x-1)^2 + (y-8)^2 = 9(x-1)^2 + 9y^2 \quad \text{translate this to} \quad (x-x_1)^2 + (y-y_1)^2 = r^2$$

$$x^2 - 2x + 1 + y^2 - 16y + 64 = 9x^2 - 18x + 9 + 9y^2$$

$$8x^2 - 16x + 8y^2 + 16y - 56 = 0 \quad (1)$$

$$x^2 - 2x + y^2 + 2y - 7 = 0$$

$$(x-1)^2 - 1 + (y+1)^2 - 1 - 7 = 0$$

$$(x-1)^2 + (y+1)^2 = 9 \quad (1)$$

$\therefore$ A circle centre $(1, -1)$, radius $\sqrt{9} = 3$. (1)

(b) distance (from centre of circle to $3 - 3i$):

$$\sqrt{(3-1)^2 + (-3-(-1))^2} = \sqrt{8} = 2.8 \dots \quad (1)$$

radius

$2.8 < 3$ so the point is within the circle

$\therefore z = 3 - 3i$ satisfies the inequality. (1)

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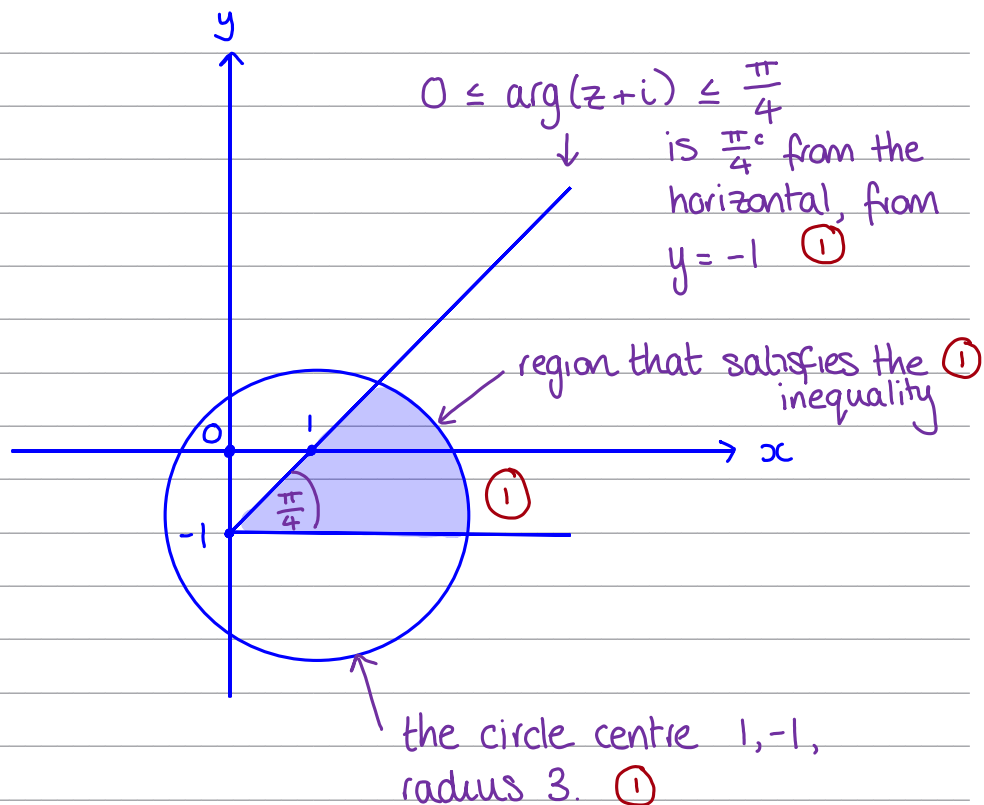
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Question 3 continued

(c)



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Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 10 marks)



4. The set $\{e, p, q, r, s\}$ forms a group, A , under the operation $*$

Given that e is the identity element and that

$$p * p = s \quad s * s = r \quad p * p * p = q$$

(a) show that

(i) $p * q = r$

(ii) $s * p = q$

(2)

(b) Hence complete the Cayley table below.

$*$	e	p	q	r	s
e	e	p	q	r	s
p	p	s	r	e	q
q	q	r	p	s	e
r	r	e	s	q	p
s	s	q	e	p	r

(2)

A spare table can be found on page 11 if you need to rewrite your Cayley table.

(2)

(c) Use your table to find $p * q * r * s$

(1)

A student states that there is a subgroup of A of order 3

(d) Comment on the validity of this statement, giving a reason for your answer.

(2)

$$(a)(i) \quad p * q = p * p * p * p \quad \leftarrow \quad p * p * p = q$$

$$= s * s \quad \leftarrow \quad p * p = s$$

$$= r \quad \textcircled{1} \quad \leftarrow \quad s * s = r$$

$$(ii) \quad s * p = p * p * p \quad \leftarrow \quad p * p = s$$

$$= q \quad \textcircled{1} \quad \leftarrow \quad p * p * p = q$$



Question 4 continued

(c) $p * q * r * s = e$ using the table, do:

- 1 $p * q = r$
- 2 $r * r = q$
- 3 $q * s = e$

number of elements



(d) The order of a subgroup must be a factor of the order of the group, according to Lagrange's Theorem. ①

As 3 is NOT a factor of 5, the student's statement is incorrect. ①

Only use this grid if you need to rewrite the Cayley table.

*	<i>e</i>	<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>
<i>e</i>					
<i>p</i>					
<i>q</i>					
<i>r</i>					
<i>s</i>					

(Total for Question 4 is 7 marks)



5. On Jim's 11th birthday his parents invest £1000 for him in a savings account.

The account earns 2% interest each year.

On each subsequent birthday, Jim's parents add another £500 to this savings account.

Let U_n be the amount of money that Jim has in his savings account n years after his 11th birthday, once the interest for the previous year has been paid and the £500 has been added.

- (a) Explain, in the context of the problem, why the amount of money that Jim has in his savings account can be modelled by the recurrence relation of the form

$$U_n = 1.02U_{n-1} + 500 \qquad U_0 = 1000 \qquad n \in \mathbb{Z}^+ \quad (3)$$

- (b) State an assumption that must be made for this model to be valid.

(1)

- (c) Solve the recurrence relation

$$U_n = 1.02U_{n-1} + 500 \qquad U_0 = 1000 \qquad n \in \mathbb{Z}^+ \quad (5)$$

Jim hopes to be able to buy a car on his 18th birthday.

- (d) Use the answer to part (c) to find out whether Jim will have enough money in his savings account to buy a car that costs £4 500

(2)

(a) U_{n-1} is the amount in the savings account $n-1$ years after Jim's 11th birthday. This gains 2% each year so is multiplied by 1.02 to give $1.02U_{n-1}$ ①

Jim's parents invest £500 for each subsequent birthday, so + 500 ①

$U_0 = 1000$ because this is the amount invested on Jim's 11th birthday. ①

(b) The interest rate stays the same each year ①

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Question 5 continued

$$(c) \quad U_n = \underbrace{1.02U_{n-1}}_{CF} + \underbrace{500}_{PS} \quad \leftarrow \text{select relevant complementary function}$$

$$U_n = c(1.02^n) + PS \quad (1) \quad U_n = c(a^n)$$

$$\lambda = 1.02\lambda + 500 \quad (1) \quad \leftarrow \text{select relevant particular solution}$$

$$g(n) = pn + q \Rightarrow \lambda = n\lambda + \mu$$

$$0.02\lambda = 500$$

$$\lambda = -25000 \quad (1)$$

$$\text{Using } U_0 = 1000:$$

$$1000 = c(1.02^0) - 25000$$

$$\therefore c = 26000 \quad (1)$$

$$U_n = 26000(1.02^n) - 25000 \quad (1) \quad (n \geq 0)$$

$$(d) \quad 18 - 11 = 7 \text{ years}$$

$$U_7 = 26000(1.02^7) - 25000 \quad (1)$$

$$U_7 = 4865.83$$

$$4865.83 > 4500 \quad (1) \quad \therefore \text{Jim will have enough money to buy the car.}$$



Question 5 continued

Lined area for writing the answer to Question 5.

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Question 5 continued

Lined area for writing the answer to Question 5.



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